

Ultrasound Physics Formula Cheat Sheet

An In-Depth Review by Dr. Alice Matthews

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Based on Edelman, "Understanding Ultrasound Physics" – All 22 Chapters

MASTER CONSTANTS

Constant	Value	Why It Matters
Speed in soft tissue	1,540 m/s = 1.54 mm/ μ s	The machine assumes this for all calculations
Wavelength at 1 MHz	1.54 mm	Divide by MHz to get λ at any frequency
Attenuation coefficient	0.5 dB/cm/MHz ($= f/2$)	Higher frequency fades faster, less penetration
13- μ s rule	13 μ s = 1 cm depth (round trip)	Converts echo time to depth on screen
Half round-trip speed	77,000 cm/s	Divide by depth (cm) to get max PRF
+3 dB / -3 dB	Intensity $\times 2$ / $\times \frac{1}{2}$	Most common dB conversion on exams
+10 dB / -10 dB	Intensity $\times 10$ / $\times \frac{1}{10}$	Combine with ± 3 dB for any dB change
Speed in PZT crystal	4–6 mm/ μ s	For transducer frequency ($3\text{--}5 \times$ soft tissue)
Imaging frequency range	2–10 MHz	Low = deep; high = shallow + better detail
Duty factor	0.2–0.5% (CW = 100%)	System listens >99% of the time
Body temperature	37°C	Bioeffects concern above 39°C
Normal hematocrit	~45%	Low hematocrit = less scattering = darker blood
Doppler shift range	20 Hz–20 kHz	Audible range (heard through speakers)
Speed of sound in water	1,480 m/s	Close to soft tissue; used in test phantoms
Rayleigh scattering	$\propto f^4$	Frequency doubles $\rightarrow$ scattering $\times 16$ (red blood cells)

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CHAPTER 1: The Basics

No formulas. Key concepts:

- **Reciprocal:** Two numbers whose product = 1 (e.g., 2 and $\frac{1}{2}$)
- **“Increase by a factor of X”** = multiply by X
- **“Decrease by a factor of X”** = divide by X

Metric Prefixes

Prefix	Symbol	Value	Complementary Pair
giga	G	10^9	giga & nano
mega	M	10^6	mega & micro
kilo	k	10^3	kilo & milli
milli	m	10^{-3}	
micro	μ	10^{-6}	
nano	n	10^{-9}	

Unit pairing rule: If frequency is in MHz, then period is in μs . If frequency is in kHz, period is in ms.

CHAPTER 2: Sound

No formulas. Key definitions:

Acoustic Variables (identify a wave as sound)

Variable	Units
Pressure	pascals (Pa)
Density	kg/cm^3
Distance (particle motion)	cm, mm

Interference

- **Constructive** (in phase): waves combine, greater amplitude
 - **Destructive** (out of phase): waves combine, lesser amplitude
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CHAPTER 3: Describing Sound Waves

Period and Frequency (Reciprocals)

$$T = \frac{1}{f} \quad f = \frac{1}{T}$$

Plain English: Period and frequency are two sides of the same coin. If you know one, just flip it to get the other. A fast vibration (high frequency) means each cycle is short (small period).

When to use: Whenever a problem gives you period and asks for frequency, or vice versa. Also the foundation for understanding why you cannot change frequency during a scan – it is locked in the moment you pick up the transducer.

- T = period, time for one cycle (s, μ s)
- f = frequency, cycles per second (Hz, MHz)
- Inversely related. Determined by **source only**. Not adjustable.
- Typical values: $T = 0.1 - 0.5 \mu$ s, $f = 2 - 10$ MHz

Sound Frequency Ranges

Type	Range
Infrasound	< 20 Hz
Audible	20 Hz to 20 kHz
Ultrasound	> 20 kHz

Power and Amplitude

$$P \propto A^2$$

- If amplitude doubles ($\times 2$), power increases $\times 4$
- If amplitude halves ($\times \frac{1}{2}$), power decreases to $\frac{1}{4}$
- Power units: watts (W). Typical: 4 – 90 mW

Intensity

$$I = \frac{P}{\text{Area}}$$

Plain English: Intensity is how concentrated the energy is. Same power through a smaller beam = higher intensity. Think of a garden hose vs. a pressure washer – same water flow, but the narrow nozzle concentrates the force.

When to use: Intensity is the key to understanding bioeffects and safety. SPTA intensity determines tissue heating. The ALARA principle is all about keeping intensity as low as possible while still getting a diagnostic image.

- I = intensity (W/cm^2). Typical: 0.01 – 300 W/cm^2
- P = power (W)
- Area = beam cross-sectional area (cm^2)

$$I \propto P \quad I \propto A^2$$

Peak-to-Peak Amplitude

$$A_{p-p} = 2A$$

Wavelength (in soft tissue)

$$\lambda \text{ (mm)} = \frac{1.54}{f \text{ (MHz)}}$$

Plain English: Wavelength is the physical length of one complete cycle in tissue. Higher frequency = shorter wavelength = better resolution but less penetration. The 1.54 comes from the speed of sound in soft tissue.

When to use: This is the gateway formula. Wavelength feeds into axial resolution, spatial pulse length, near zone length, and beam divergence. If you know the transducer frequency, you can derive almost everything else.

- λ and f are **inversely related** (in the same medium)
- Determined by **BOTH source AND medium** (the only parameter determined by both)

- Not adjustable. Typical: 0.15 – 0.8 mm

Frequency	Wavelength in Soft Tissue
1 MHz	1.54 mm
2 MHz	0.77 mm
3 MHz	0.51 mm
5 MHz	0.31 mm
10 MHz	0.15 mm

Propagation Speed

$$c = f \times \lambda$$

- c = propagation speed (m/s)
- Determined by **medium only**. Not adjustable.
- Speed is **UNRELATED** to frequency.

$$c_{\text{soft tissue}} = 1,540 \text{ m/s} = 1.54 \text{ mm}/\mu\text{s} = 1.54 \text{ km/s}$$

Speed of Sound in Biologic Media

Tissue	Speed (m/s)
Lung	500
Fat	1,450
Soft Tissue	1,540
Liver	1,560
Blood	1,560
Muscle	1,600
Tendon	1,700
Bone	3,500

Other materials: Air = 330, Water = 1,480, Metals = 2,000 – 7,000

Rule: Solids = fastest, Liquids = slower, Gases = slowest

How Medium Properties Affect Speed

Property	Relationship to Speed
Stiffness	Directly related ($\uparrow$ stiffness = $\uparrow$ speed)
Density	Inversely related ($\uparrow$ density = $\downarrow$ speed)
Elasticity	Inversely related (elastic = not stiff)
Compressibility	Inversely related (compressible = not stiff)

Stiffness synonyms: bulk modulus, elasticity, compressibility

Parameter Summary (Table 3.5)

Parameter	Determined By	Adjustable	Units
Period	source	No	s, μs
Frequency	source	No	Hz, MHz
Amplitude	source	Yes	Pa, dB
Power	source	Yes	W
Intensity	source	Yes	W/cm ²
Wavelength	both	No	mm
Speed	medium	No	m/s

Key Relationships

Pair	Relationship
f and T	Inversely (reciprocals)
A and P	Directly
A and I	Directly ($I \propto A^2$)
P and I	Directly
λ and f	Inversely (same medium)
Stiffness and c	Directly
f and c	UNRELATED
λ and I	UNRELATED
P and f	UNRELATED

CHAPTER 4: Describing Pulsed Waves

Pulse Duration

$$PD (\mu s) = n \times T (\mu s)$$

$$PD (\mu s) = \frac{n}{f (\text{MHz})}$$

- PD = pulse duration (time, μs). Typical: 0.5 – 3.0 μs
- n = number of cycles per pulse. Typical: 2 – 4
- Determined by **source**. Not adjustable.

Spatial Pulse Length

$$SPL (\text{mm}) = n \times \lambda (\text{mm})$$

Plain English: SPL is how long the pulse is in space – like measuring the length of a train. Fewer cycles or shorter wavelengths make a shorter pulse, which means better axial resolution.

When to use: SPL is the direct input to axial resolution ($AR = SPL/2$). Shorter pulse = ability to distinguish two structures that are close together along the beam's path.

- SPL = spatial pulse length (distance, mm). Typical: 0.1 – 1.0 mm
- Determined by **BOTH source AND medium**. Not adjustable.

Key distinction: PD = time (how long the train takes to pass) SPL = distance (how long the train is end-to-end)

PRP and PRF (Reciprocals)

$$PRP \times PRF = 1$$

$$PRP = \frac{1}{PRF} \quad PRF = \frac{1}{PRP}$$

- PRP = pulse repetition period (ms). Typical: 0.1 – 1.0 ms
- PRF = pulse repetition frequency (Hz). Typical: 1 – 10 kHz
- Both **adjustable** (by changing imaging depth)

PRP is **UNRELATED** to Period. PRF is **UNRELATED** to Frequency.

Duty Factor

$$DF (\%) = \frac{PD}{PRP} \times 100$$

Plain English: Duty factor tells you what fraction of time the transducer is actually transmitting vs. listening. At 0.2 – 0.5%, the system is quiet over 99% of the time. Only CW Doppler transmits continuously ($DF = 100\%$).

When to use: Duty factor connects to bioeffects – higher duty factor means more energy delivered to tissue. Also a common exam question: “Which has a higher duty factor, pulsed or CW?”

- Unitless (percentage). Typical: 0.2% – 0.5%
- CW duty factor = 100%
- Adjustable (via imaging depth)

Shallow vs. Deep Imaging

	Shallow	Deep
Listening	Less	More
PRP	Shorter	Longer
PRF	Higher	Lower
DF	Higher	Lower

CHAPTER 5: Intensities

Spatial Measurements

- **Spatial Peak (SP):** intensity at the center of the beam (maximum)
- **Spatial Average (SA):** average intensity across the beam’s cross-section
- I_{SP} is always $> I_{SA}$

Beam Uniformity Coefficient

$$\frac{SP}{SA} = \frac{I_{SP}}{I_{SA}} \geq 1$$

Temporal Measurements (largest to smallest)

$$I_{TP} > I_m > I_{PA} > I_{TA}$$

I_{TA} can be up to $500\times$ lower than the others (system is off $\sim 99.8\%$ of the time)

Combined Measurements (largest to smallest)

$$SPTP > I_m > SPPA > SPTA > SATA$$

CW Equivalences

For continuous wave: $I_{PA} = I_{TA}$, so $SPTA = SPPA$ and $SATA = SAPA$

Key Facts

- All intensities in W/cm^2
 - $SPTA$ is the most relevant intensity for tissue heating and bioeffects
 - $SPTP$ is always the highest; $SATA$ is always the lowest
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CHAPTER 6: Interaction of Sound and Media

Decibel Notation

dB Change	Intensity Change
+3 dB	$\times 2$ (doubles)
+6 dB	$\times 4$
+9 dB	$\times 8$
+10 dB	$\times 10$
+20 dB	$\times 100$
-3 dB	$\times \frac{1}{2}$
-6 dB	$\times \frac{1}{4}$
-10 dB	$\times \frac{1}{10}$
-20 dB	$\times \frac{1}{100}$

Decibels are **logarithmic**, **relative**, and **additive**.

Attenuation

$$\text{Total Attenuation (dB)} = \alpha \text{ (dB/cm)} \times d \text{ (cm)}$$

- α = attenuation coefficient
- d = distance (path length)

Attenuation Coefficient in Soft Tissue

$$\alpha \text{ (dB/cm)} = \frac{f \text{ (MHz)}}{2}$$

Plain English: For every centimeter the beam travels in soft tissue, it loses half the frequency number in decibels. A 10 MHz beam loses 5 dB per centimeter – that adds up fast, which is why high-frequency probes cannot image deep structures.

When to use: Calculating total signal loss over a path. Multiply this coefficient by the distance to find how many dB the beam has lost. Also determines why you pick a low-frequency probe for deep targets and a high-frequency probe for superficial ones.

Rule of thumb: **0.5 dB per cm per MHz**

Examples: - 2 MHz: $\alpha = 1 \text{ dB/cm}$ - 5 MHz: $\alpha = 2.5 \text{ dB/cm}$ - 7 MHz traveling 5 cm: total = $3.5 \times 5 = 17.5 \text{ dB}$

Attenuation Ranking (lowest → highest)

water < blood/fluids < fat < soft tissue < muscle < bone/lung < air

Half-Value Layer (HVL)

- Distance that reduces intensity to $\frac{1}{2}$ its original value (= 3 dB of attenuation)
- Units: cm. Typical: 0.25 – 1.0 cm
- High frequency or high-attenuation media = thin HVL
- Low frequency or low-attenuation media = thick HVL

Acoustic Impedance

$$Z = \rho \times c$$

Plain English: Impedance is a tissue's resistance to sound passing through it. Dense, stiff tissues (like bone) have high impedance. When two tissues have very different impedances, a strong reflection occurs at their boundary – that is what creates the bright echoes you see on the image.

When to use: Impedance is the input to the reflection coefficient (IRC). Whenever you see a bright boundary on an image, it means there is a large impedance mismatch. Also explains why gel is needed – air has extremely different impedance from skin.

- Z = acoustic impedance (Rayls)
- ρ = density (kg/m^3)
- c = propagation speed (m/s)
- Typical biologic media: 1.25 – 1.75 MRayls
- Determined by **medium only**. Calculated, not measured.

Intensity Reflection Coefficient (Normal Incidence)

$$IRC (\%) = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1} \right)^2 \times 100$$

Plain English: This tells you what percentage of the sound bounces back at a boundary. The bigger the impedance difference, the stronger the reflection. At most soft tissue interfaces, less than 1% reflects – plenty of sound continues deeper. But at a tissue-air or tissue-bone boundary, nearly everything reflects, creating shadowing.

When to use: Predicting echo brightness at boundaries, explaining shadowing behind bone or gas, and understanding why coupling gel eliminates the skin-air interface.

- Soft tissue boundaries: typically $\leq 1\%$ reflected

Intensity Transmission Coefficient

$$ITC (\%) = 100\% - IRC (\%)$$

Conservation of Energy

$$I_{\text{incident}} = I_{\text{reflected}} + I_{\text{transmitted}}$$

Snell's Law (Refraction)

$$\frac{\sin \theta_t}{\sin \theta_i} = \frac{c_2}{c_1}$$

Plain English: When sound hits a boundary at an angle and the two tissues have different speeds, the beam bends. Faster tissue on the other side = beam bends away from the perpendicular. This is exactly like light bending when it enters water. If the beam hits straight on (perpendicular), no bending occurs regardless of speed difference.

When to use: Explaining refraction artifacts (like the double-image of the aorta through the rectus muscles). Also important: refraction requires both oblique incidence AND different speeds – if either condition is missing, no refraction.

- θ_i = incident angle
- θ_t = transmission angle
- **Two conditions required:** (1) oblique incidence AND (2) different speeds
- $c_2 > c_1$: beam bends **away** from normal ($\theta_t > \theta_i$)
- $c_2 < c_1$: beam bends **toward** normal ($\theta_t < \theta_i$)
- $c_2 = c_1$: no refraction

Reflection at Oblique Incidence

$$\theta_{\text{reflection}} = \theta_{\text{incidence}}$$

Rayleigh Scattering

$$\text{Rayleigh scattering} \propto f^4$$

- Frequency doubles $\rightarrow$ scattering increases $\times 16$ ($2^4 = 16$)
 - Occurs when structures $\ll \lambda$ (e.g., red blood cells)
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CHAPTER 7: Range Equation

Depth from Time-of-Flight

$$d \text{ (mm)} = \frac{1.54 \text{ mm}/\mu\text{s} \times t \text{ (}\mu\text{s)}}{2}$$

Plain English: The machine measures how long it takes an echo to return, then divides by two (because the sound traveled there and back). This is how every dot gets placed on the screen. It assumes sound travels at 1.54 mm per microsecond in all tissue.

When to use: Any problem that gives you a round-trip time and asks for depth, or vice versa. Also the basis for understanding speed artifacts – if the actual tissue speed is not 1.54, the dot gets placed at the wrong depth.

- d = reflector depth
- t = go-return time
- Division by 2 accounts for the round trip

The 13-Microsecond Rule

$$13 \mu\text{s of round-trip time} = 1 \text{ cm of depth in soft tissue}$$

Plain English: A quick shortcut. For every centimeter of depth, the sound needs 13 microseconds to make the round trip. Imaging at 10 cm deep? The system waits 130 microseconds before sending the next pulse.

Time	Depth	Total Distance
13 μs	1 cm	2 cm
26 μs	2 cm	4 cm
39 μs	3 cm	6 cm
130 μs	10 cm	20 cm

PRP from Imaging Depth

$$PRP (\mu s) = d (\text{cm}) \times 13 \mu s/\text{cm}$$

PRF from Imaging Depth

$$PRF (\text{Hz}) = \frac{77,000}{d (\text{cm})}$$

Plain English: The deeper you image, the longer the system waits for echoes, so it can send fewer pulses per second. At 10 cm depth, the maximum PRF is 7,700 Hz. At 20 cm, only 3,850 Hz.

When to use: Connects depth to frame rate and to aliasing in Doppler. Deeper imaging = lower PRF = lower Nyquist limit = more chance of aliasing.

- $77,000 = \frac{154,000}{2}$ (half the round-trip speed in cm/s)

CHAPTER 8: Axial Resolution

$$AR (\text{mm}) = \frac{SPL (\text{mm})}{2}$$

$$AR (\text{mm}) = \frac{n \times \lambda (\text{mm})}{2}$$

In soft tissue:

$$AR (\text{mm}) = \frac{0.77 \times n}{f (\text{MHz})}$$

Plain English: Axial resolution is the ability to tell apart two structures that are stacked on top of each other along the beam's path. It equals half the pulse length. Shorter pulse = better resolution. This is the best resolution the system has – always better than lateral or elevational.

When to use: Choosing a transducer for fine detail. Higher frequency = shorter wavelength = shorter SPL = better axial resolution. But you trade off penetration depth. Also remember: axial resolution does NOT change with depth (unlike lateral).

- **Lower number = BETTER resolution**
- Typical: 0.1 – 1.0 mm
- Not adjustable by sonographer
- Synonyms: Longitudinal, rAnge, Radial, Depth (LARRD)

Better axial resolution: shorter *SPL*, higher *f*, fewer cycles per pulse

CHAPTER 9: Transducers

Piezoelectric Element Dimensions

$$\text{PZT thickness} = \frac{\lambda_{\text{PZT}}}{2}$$

$$\text{Matching layer thickness} = \frac{\lambda_{\text{matching}}}{4}$$

Transducer Frequency (Pulsed)

$$f \text{ (MHz)} = \frac{c_{\text{PZT}} \text{ (mm}/\mu\text{s)}}{2 \times \text{thickness (mm)}}$$

- Speed of sound in PZT: 4 – 6 mm/ μs (3 – 5 $\times$ faster than soft tissue)
- Thinner PZT = higher frequency (inversely related)
- CW transducer: frequency = electrical signal frequency (not PZT thickness)

Quality Factor

$$Q = \frac{f_{\text{main}}}{\text{Bandwidth}}$$

$$\text{Bandwidth} = f_{\text{highest}} - f_{\text{lowest}}$$

Type	Bandwidth	Q
Imaging (damped)	Wide	Low
Therapeutic/CW (undamped)	Narrow	High

CHAPTER 10: Sound Beams

Near Zone Length (Focal Depth) – Unfocused Disc

$$NZL \text{ (cm)} = \frac{D^2 \text{ (mm)} \times f \text{ (MHz)}}{61.6}$$

$$NZL \text{ (cm)} = \frac{D^2 \text{ (mm)}}{4 \times \lambda \text{ (mm)}}$$

Plain English: The near zone is where the beam naturally converges to its narrowest point (the focus). A bigger transducer face or higher frequency pushes that focal point deeper. Beyond the focus, the beam spreads out and image quality drops.

When to use: Understanding why a larger transducer can image deeper with a tight beam, and why small phased-array probes (cardiac) have a short near zone. The focus is where lateral resolution is best.

- D = transducer diameter
- Larger diameter = deeper focus
- Higher frequency = deeper focus

Beam Divergence Angle (Far Field)

$$\sin \theta_{\text{div}} = \frac{1.22 \lambda}{D}$$

- Larger diameter = less divergence
- Higher frequency = less divergence

Beam Width at Key Locations

Location	Beam Width
Transducer face	$= D$
Focus (end of near zone)	$= \frac{D}{2}$
$2 \times NZL$	$= D$
Beyond $2 \times NZL$	$> D$

Lateral Resolution

LR = beam diameter at that depth

- Best at the focus (narrowest beam)
 - Changes with depth (unlike axial resolution)
 - Synonyms: Lateral, Angular, Transverse, Azimuthal (LATA)
 - Axial resolution is always **BETTER** than lateral resolution
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CHAPTER 11: Display Modes

Mode	Horizontal	Vertical	Shows
A-mode	Depth	Amplitude	Spikes
B-mode	Depth	–	Brightness dots
M-mode	Time	Depth	Motion over time

- M-mode sampling rate = PRF (highest temporal resolution)
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CHAPTER 12: Two-Dimensional Imaging

Array Element Sizes

Array Type	Element Width	Elements	Footprint
Linear phased	$\frac{\lambda}{4}$ to $\frac{\lambda}{2}$	100-300	~ 1 cm
Linear sequential	$\sim \lambda$	120-250	up to 10 cm
Convex	$\sim \lambda$	120-250	up to 10 cm

Resolution Hierarchy

Axial (best) > Lateral > Elevational (worst)

Phased Array Timing

- **Slope** in spike line = beam steering direction
 - **Curvature** in spike line = beam focusing
 - Greater curvature = shallower focus
 - Time differences between spikes: ~ 10 ns
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CHAPTER 13: Real-Time Imaging

Frame Rate

$$\text{Frame Rate} \times T_{\text{frame}} = 1$$

$$T_{\text{frame}} = \text{Pulses per Frame} \times PRP$$

$$PRF = \text{Lines per Frame} \times \text{Frame Rate}$$

$$\text{Pulses per Frame} = \text{Scan Lines} \times \text{Focal Points per Line}$$

Factors Affecting Frame Rate

$\uparrow$ Frame Rate	$\downarrow$ Frame Rate
Shallow depth	Deep depth
Single focus	Multi-focus
Narrow sector	Wide sector
Low line density	High line density

TV Display

- 525 lines per frame, interlaced
 - Odd field: $\frac{1}{60}$ s, Even field: $\frac{1}{60}$ s
 - Frame rate: $\frac{1}{30}$ s = 30 Hz
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CHAPTER 14: Pulsed Echo Instrumentation

Receiver Functions (in order)

1. Amplification
2. Compensation (TGC/DGC)
3. Compression
4. Demodulation
5. Reject

Signal-to-Noise Ratio

$$SNR = \frac{\text{Signal}}{\text{Noise}}$$

- $\uparrow$ output power = **improves** SNR
- $\uparrow$ receiver gain = **no change** in SNR (amplifies both equally)

ALARA Principle

As Low As Reasonably Achievable

- Image too dark $\rightarrow$ increase **receiver gain FIRST** (no added exposure)
- Image too bright $\rightarrow$ decrease **output power FIRST** (reduces exposure)

Output Power vs. Receiver Gain

	Output Power	Receiver Gain
Brightness	Changes	Changes
SNR	Improves	No change
Patient exposure	Increases	No change

CHAPTER 15: Displays and Image Storage

Gray Shades from Bit Depth

$$\text{Gray Shades} = 2^n$$

Bits (n)	Shades
1	2
4	16
8	256

More bits = more gray shades = better contrast resolution

CHAPTER 16: Dynamic Range

Dynamic Range in Decibels (Voltage Ratio)

$$DR \text{ (dB)} = 20 \times (\text{number of zeros in voltage ratio})$$

Voltage Ratio	Dynamic Range
10 : 1	20 dB
100 : 1	40 dB
1,000 : 1	60 dB
10,000 : 1	80 dB
100,000 : 1	100 dB

Compression

$$DR_{\text{compressed}} = DR_{\text{original}} - \text{Compression (dB)}$$

Dynamic Range Through the Imaging Chain

Component	Dynamic Range
Transducer	120 dB
Receiver	100-120 dB
Scan converter	40-50 dB
Display	20-30 dB

- **Narrow** DR = fewer gray shades = high contrast (black/white)
 - **Wide** DR = many gray shades = low contrast (grayscale)
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CHAPTER 17: Harmonics and Contrast Agents

Harmonic Frequency

$$f_{\text{harmonic}} = 2 \times f_{\text{fundamental}}$$

Example: Transmit 2 MHz, receive harmonics at 4 MHz

Mechanical Index

$$MI = \frac{P_{\text{neg}}}{\sqrt{f}}$$

Plain English: MI predicts the risk of cavitation – the formation and collapse of gas bubbles in tissue. High pressure and low frequency both increase the risk. The display shows MI in real time so you can monitor safety during scanning.

When to use: Safety monitoring, especially with contrast agents (microbubbles). Low MI (< 0.1) preserves bubbles for imaging. High MI (> 1.0) destroys them. Also relevant for obstetric scanning, where you want to keep MI low.

- P_{neg} = peak negative (rarefaction) pressure
- f = frequency
- MI is **unitless**
- Higher MI : more pressure, lower frequency
- Lower MI : less pressure, higher frequency

MI Thresholds (Contrast Agents)

MI	Microbubble Behavior
< 0.1	Linear, no harmonics
$0.1 - 1.0$	Resonance, strong harmonics
> 1.0	Bubble disruption

CHAPTER 18: Hemodynamics

Pressure-Flow-Resistance (Ohm's Law Analogy)

$$\Delta P = Q \times R$$

Plain English: Blood flow works like electrical circuits. Pressure difference drives the flow, and resistance opposes it. A stenosis increases resistance, so you need more pressure to push the same flow through. This is why blood speeds up at a narrowing – the body maintains flow by increasing velocity through the smaller opening.

When to use: Understanding why stenoses cause velocity increases and pressure drops. Also explains why collateral pathways develop – blood finds lower-resistance routes around an obstruction.

Fluid	Electrical
Pressure (ΔP)	Voltage (V)
Flow (Q)	Current (I)
Resistance (R)	Resistance (R)

Hydrostatic Pressure

$$P_{\text{measured}} = P_{\text{circulatory}} + P_{\text{hydrostatic}}$$

- Below heart: $P_{\text{hydrostatic}} > 0$ (higher measured BP)
- Above heart: $P_{\text{hydrostatic}} < 0$ (lower measured BP)
- At heart level: $P_{\text{hydrostatic}} = 0$
- Supine patient: $P_{\text{hydrostatic}} = 0$ everywhere

Reynolds Number Thresholds

$$Re < 1,500 \implies \text{Laminar flow}$$

$$Re > 2,300 \implies \text{Turbulent flow}$$

CHAPTER 19: Doppler

Doppler Shift (Basic)

$$f_d = f_r - f_t$$

- f_d = Doppler shift (Hz)
- f_r = reflected frequency
- f_t = transmitted frequency
- Positive f_d = flow **toward** transducer
- Negative f_d = flow **away** from transducer
- Doppler shifts fall in audible range: 20 Hz to 20 kHz

The Doppler Equation

$$f_d = \frac{2v f_t \cos \theta}{c}$$

Plain English: The Doppler shift is the frequency change caused by moving blood. Faster blood = bigger shift. The cosine factor accounts for the angle between your beam and the blood flow direction. At 0 degrees (beam parallel to flow), you get the maximum shift. At 90 degrees, cosine is zero and you get no Doppler signal at all.

When to use: This is the core of all Doppler imaging. It explains why angle matters so much – always keep the Doppler angle under 60 degrees. At 60 degrees, you only detect half the true velocity. It also shows why higher transmit frequency produces a larger shift (but less penetration).

- v = blood velocity (m/s)
- f_t = transmitted frequency (Hz)
- θ = angle between beam and flow direction
- c = speed of sound in soft tissue (1,540 m/s)
- The factor of 2 accounts for two Doppler shifts (transmit and receive)

Angle Correction

$$v_{\text{measured}} = v_{\text{true}} \times \cos \theta$$

Angle (θ)	$\cos \theta$	Velocity Measured
0°	1.0	100% (toward)
30°	0.87	87%
60°	0.5	50%
90°	0.0	NO SIGNAL
180°	-1.0	100% (away)

Nyquist Limit (Aliasing)

$$f_{\text{Nyquist}} = \frac{PRF}{2}$$

Plain English: The Nyquist limit is the highest Doppler shift that pulsed Doppler can correctly measure. It equals half the PRF. If blood is moving fast enough to produce a shift above this limit, the signal wraps around (aliasing) – the display shows flow in the wrong direction.

When to use: Whenever you see aliasing on a color or spectral Doppler display. To fix it: increase the PRF (raise the scale), use a lower frequency transducer, decrease depth, or switch to CW Doppler (which has no Nyquist limit because it transmits continuously).

- Aliasing occurs **ONLY** with pulsed Doppler, **NEVER** with CW

Five techniques to reduce aliasing:

1. Increase scale (increase PRF)
2. Use lower frequency transducer
3. Use shallower sample volume
4. Switch to CW Doppler
5. Baseline shift (cosmetic only)

Resistivity Index

$$RI = \frac{V_{\text{max}} - V_{\text{min}}}{V_{\text{max}}}$$

Plain English: RI measures how much flow resistance exists downstream. High RI (close to 1.0) means high resistance – very little diastolic flow. Low RI means the vessel feeds an organ that needs constant blood supply (like the brain or kidney).

Pulsatility Index

$$PI = \frac{V_{\max} - V_{\min}}{V_{\text{mean}}}$$

Plain English: PI is similar to RI but uses the mean velocity in the denominator, making it more sensitive to waveform shape. PI can exceed 1.0 (unlike RI). Both RI and PI are angle-independent because they are ratios – the cosine factor cancels out.

When to use (RI and PI): Evaluating downstream resistance in organs. Commonly applied in renal, hepatic, and obstetric Doppler. An abnormally high RI in a transplant kidney, for example, may indicate rejection.

- Unitless. Unaffected by Doppler angle.

Color vs. Spectral Doppler

	Color Doppler	Spectral Doppler
Reports	Mean velocity	Peak velocity
Analysis	Autocorrelation	FFT

CHAPTER 20: Artifacts

Six Imaging Assumptions (violations create artifacts)

1. Sound travels in a straight line
2. Sound travels directly to a reflector and back
3. Sound travels at exactly 1,540 m/s
4. Reflections come only from the main beam axis
5. The imaging plane is very thin
6. Reflection strength relates to tissue characteristics

Speed Error

$c_{\text{actual}} > 1,540 \text{ m/s} \implies$ distances UNDERESTIMATED (too shallow)

$c_{\text{actual}} < 1,540 \text{ m/s} \implies$ distances OVERESTIMATED (too deep)

CHAPTER 21: Quality Assurance

No formulas. Key measurements:

- AIUM 100 mm test object: speed = 1.54 km/s
- Tissue equivalent phantom: speed = 1,540 m/s

QA measures: sensitivity, dead zone, registration accuracy, range accuracy, horizontal calibration, axial resolution, lateral resolution, uniformity, slice thickness

CHAPTER 22: Bioeffects

Thermal Index (TI)

TI = predictor of maximum temperature increase (unitless)

- TIS = soft tissue thermal index
- TIB = bone thermal index (bone at/near focus)
- TIC = cranial bone thermal index (bone in near field)
- Best estimate of in vivo temperature elevation
- Maximal heating related to $SPTA$ intensity

Mechanical Index (MI)

$$MI = \frac{P_{neg}}{\sqrt{f}}$$

- Best indicator of **cavitation** risk. Unitless.

Bioeffects Thresholds

Threshold	Value
No confirmed bioeffects	$< 2^{\circ}\text{C}$ rise for < 50 hours
No fetal defects observed	$< 39^{\circ}\text{C}$ ($< 2^{\circ}$ above normal)
Normal body temperature	37°C

Cavitation Types

- **Stable:** lower MI , oscillating bubbles, microstreaming
- **Transient:** higher MI , bubble bursting, shock waves
- Transient threshold is only 10% above stable threshold

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